

Stress and Deflection Analysis of Composite Cantilever Beam using Software— A New Trend Approach

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Abstract—Composites materials components have been use majorly in the industry form many years because composites executes much better than the comparable homogenous isotropic materials. Advanced composite materials like fiber reinforced composite are majorly used in aerospace industry. The advantages of composite such as high specific strength, high strength to weight ratio and stiffness, good corrosion resistance, and lower thermal expansion make them an ultimate preference in aircraft structures and other applications. The designer's freedom to choose from various basic constituents of Composite materials to achieve properties for a particular application/mechanisms makes them attractive option for design. In this work our main focus is on calculating the stress and deflection of composite cantilever beam which can be used in many industrial work. For that composite beam made up of AS4 3501-6 Graphite-Epoxy is chosen for this work. The research carried out in this project will enable to determine the beam strength due to Transverse loads. In this stress and deflection are calculated with analytical solver developed in the Matlab software and its results are validated with Finite Element Analysis software.

Keywords: Cantilever Beam, Composites, Isotropic materials, Stress, Deflection, Stiffness, Matlab, FEA.

1. INTRODUCTION

Designing a composite material for mechanical equipment consists of an optimization process in which designer's always has to check or refer certain objectives for example strength, deflection, weight, wear, corrosion, etc. that is depending on the requirements. Fiber-reinforced composites are very much in demand by the industry because of their high specific stiffness/strength ratios especially for applications where weight reduction is critical. By using composite materials in practice, weight of a structure can be reduced significantly. Even more reduction in weight is also possible by optimizing the material system itself such as fiber orientations, ply thickness, stacking sequence, etc. Since many research guides trying to make a best use of composites either by minimizing the laminate thickness thus reducing the weight, or by maximizing static strength of laminates for a given thickness.

The structural elements having one dimension many times greater than its other dimensions can be a rod, a bar, a column, or a beam. This is preferably depends on the loading conditions. A beam is a structural member mainly undergoes to bending. The terms rod (or bar) and column are for those members that are primarily undergoes to axial tension and compression, respectively.

Beams are one of the primary fundamental structural or mechanical components. Composite beams are lightweight structures that can be found in many diverse applications including aerospace, submarine, medical equipment, automotive and construction industries. Constructional Buildings, steel framed structure components and bridges are examples of beam applications in civil engineering. In these applications, beams exist as structural elements or components supporting the whole structure. In addition, the whole structure can be modeled at a preliminary level as a beam. For example, a very high/tall rise building can be modeled as a cantilever beam, whereas bridge modeled as a simply supported cantilever beam.

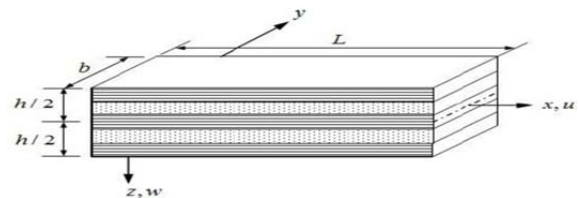


Fig. 1: Composite Beam laminates

Various researches have been done in composite materials and advanced material are coming up in the market. Mr. Nitin Jahuri, Mr. Harishchandra Thakur, Mr. Raghavendra Mishra [1] presented paper on Stress analysis in FRP composites. In this they explain that as deformation is reduced in composite laminates, on the other hands number of layers must be increased, which results in increase of von-misses stress.

Fibres are the main constituents which are responsible for strength of a composite laminate & along with fibre orientation, play an important role on its load bearing capacity. It can be inferred based on analysis that, cross-ply configuration $[0^0/90^0]$ has good load bearing capacity.

Mr.Wu Zhen, Mr.Chen Wanji [2] presented paper on Stress analysis of laminated composite plates with a circular hole according to a single-layer higher-order model. In this they conclude that single-layer higher-order model for predicting the stresses at curved free boundaries. They proposed model and the methodology of the discrete Kirchhoff plate bending element, a triangular finite element is also presented of laminated composite plates subjected to in plane loading.

Mr. Zhangxin Guo, Mr. Xiaoping Han, Mr. Xiping Zhu [3] presented a paper on Finite Element Analysis of Inter-laminar Stresses for Composite Laminates Stitched around a Circular Hole. In this they conclude that for the case of the double stitching reinforcement, the maximum inter-laminar stresses are lower than that of the single stitching reinforcement. The distribution of the inter-laminar stresses around the hole is related with layer design and there exist different transform point (changing from positive to negative values) of inter-laminar stresses around the hole.

Mr.Mehdi Hajianmaleki and Mohammad S. Qatu[4] presented paper on Mechanics of Composite Beams. In this different approaches for static and dynamic analysis of composite beams were studied and a modified FSDT model for various laminate couplings and shear deformation and rotary inertia was validated. The method was verified using 3D FEM model. The results showed good accuracy of the model for rectangular beams in static analysis for laminates having bending-twisting coupling and in dynamic analysis for all kinds of laminates. This model gives an accurate way for calculation of the natural frequencies of beams and shafts with arbitrary laminate for researchers.

2. PROBLEM FORMULATION

As the Beam being a primary structural element or primary machine components it has several uses in industries and beam is a structural member mainly undergoes to bending. So main objective of this research work is to find the bending stress and deflection of a composite beam. A cantilevered laminated composite beam made up of AS4 3501-6 Graphite-Epoxy has following layup of $[0/90/+45/-45]_s$. A uniform load of 200N is applied at free end of beam. Assume that each ply is 0.125mm.

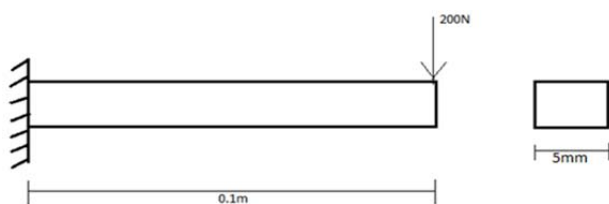


Fig. 2: Cantilever beam with point load at free end

Selection of Material

- Fiber Material = Carbon fiber (unidirectional lamina)
- Matrix Material = Epoxy resin (unidirectional lamina)

Material Properties

Table 1: Micromechanics Properties

Properties	Notation	Value
Fiber volume fraction	v_f	0.6
Matrix volume fraction	v_m	0.4
Youngs modulus of Fiber	E_f	225 GPa
Youngs modulus of Matrix	E_m	4.2 GPa
Poisson's Ratio for Fiber	μ_f	0.2
Poisson's Ratio for Matrix	μ_m	0.34
Shear Modulus for Fiber	G_f	15 GPa
Shear Modulus for Matrix	G_m	1.567 GPa
Transverse Shear Modulus for Fiber	$E_f \text{ trans}$	15 GPa
Transverse Shear Modulus for Matrix	$E_m \text{ trans}$	4.2 GPa

Specification

- Layup $[0/90/+45/-45]$, 2) Number of layers = 8
- Beam length = 100mm, Width = 5mm
- Point load at free end = 200N
- Each ply thickness = 0.125mm

3. ANALYSIS METHODOLOGY

A. Analytical Analysis

For calculating the combine matrix and fiber properties of composite lamina micro-mechanics calculations are formulated in analytical solver from that, values of single composite lamina are calculated.

Calculation of ABD matrix

The ABD matrix is a 6x6 matrix that serves as a connection between the applied loads and the associated strains in the laminate. It essentially defines the elastic properties of the entire laminate.

$$\overline{Q}_{ij} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}$$

As the ply sequence is from bottom to top the distance of k^{th} ply top and ply bottom from the mid plane is given by

$$Z_k^{\text{bot}} = - \left[\left(\frac{Nol}{2} \right) - (k-1) \right] * t$$

$$Z_k^{\text{top}} = Z_k^{\text{bot}} + t$$

Then ABD matrix is given as

Extension Stiffness Matrix $[A_{ij}]$

$$A_{ij} = \sum_{k=1}^n \{Q_{ij}\}_n (Z_k - Z_{k-1})$$

Extension-Bending coupling Stiffness Matrix $[B_{ij}]$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n \{Q_{ij}\}_n (Z_k^2 - Z_{k-1}^2)$$

Bending Stiffness Matrix $[D_{ij}]$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n \{Q_{ij}\}_n (Z_k^3 - Z_{k-1}^3)$$

Assembled a ABD Matrix as

$$ABD = \begin{bmatrix} A & B \\ B & D \end{bmatrix}$$

By combination of equations of A_{ij} , B_{ij} and D_{ij} we can calculate the ply level stress and strain as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{Bmatrix}$$

B. FEM Analysis

The values obtained from the analytical analysis are given to the FEM software and following results are obtained with ANSYS R17.0 software

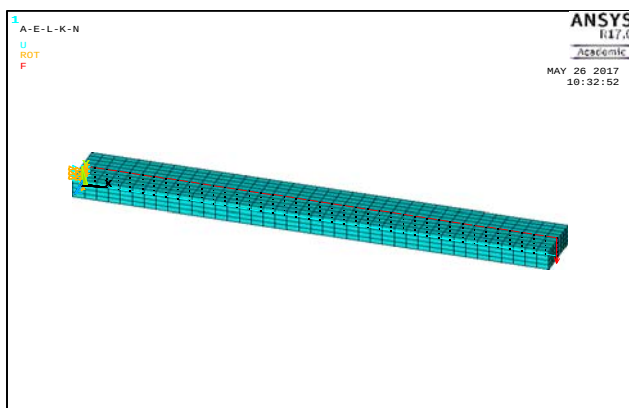


Fig. 3 Beam loading condition

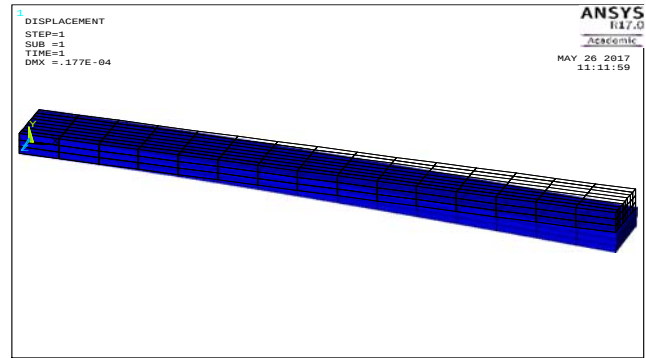


Fig. 4: Deflection of beam

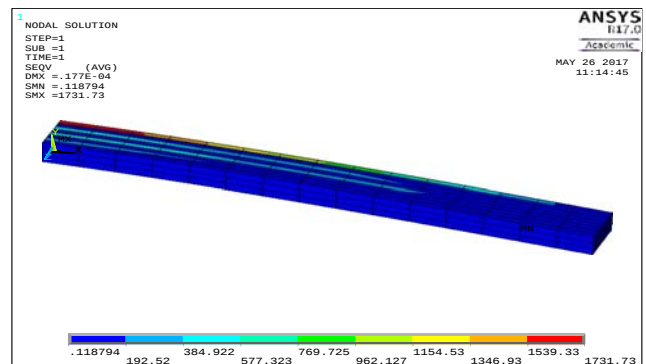


Fig. 5: Von misses stress

C. Failure Criteria Analysis by Helius Composite

For this analysis of composite Beam Maximum stress Failure criteria is used to calculate the failure behaviour of composite Beam

For this Autodesk Helius Composite Software is used it gives results as

First Ply Failure

Transverse Failure in Ply: 8

Failure Index = 4.65121E+04

Safety Factor = 2.14998E-05



Fig. 6: Progressive Failure of Beam

4. RESULT AND DISCUSSION

For Stress and Deflection analysis of composite beam a composite analytical solver is develop in Excel sheet and all values are calculated are compared with FEM Analysis.

After that a MATLAB Code for calculating the same values are developed. The results are tabulated as follows

Table 2: Results

Properties	Analytical Excel sheet	Matlab value	FEM Value	% Error
Deflection (mm)	1.859	1.823	1.77	4.78
Stress (MPa)	1780.56	1760.1	1731	2.75

These analytical results by Excel sheet which is validated by Autodesk Heliu Composite Software. This software is mainly used for a analysis of Composite structure. The stress and strains at top and bottom surface of each ply are calculated The Analytical results for stress and strains at ply level are as below

Table 3: Ply level Strainvalues in MPa (Analytical)

Ply No	Position	ϵ_1	ϵ_2	γ_{12}
1	Bottom	-0.27897	0.029240	0.08390
1	Top	-0.20923	0.021930	0.06293
2	Bottom	0.02193	-0.20922	-0.06293
2	Top	0.01462	-0.13948	-0.04195
3	Bottom	-0.04146	-0.08341	0.15410
3	Top	-0.02073	-0.04170	0.07705
4	Bottom	-0.04171	-0.02072	-0.07705
4	Top	4.12E-18	1.90E-18	1.4E-17
5	Bottom	4.12E-18	1.90E-18	1.4E-17
5	Top	0.041705	0.020727	0.07705
6	Bottom	0.020728	0.041705	-0.07705
6	Top	0.041456	0.083410	-0.15410
7	Bottom	-0.01462	0.139486	0.041954
7	Top	-0.02193	0.209229	0.062931
8	Bottom	0.209229	-0.02193	-0.06293
8	Top	0.278973	-0.02924	-0.08390

Table 4: Ply Level Stress values in MPa (Analytical)

Ply No	Position	σ_1	σ_2	τ_{12}
1	Bottom	-38232.9	-402.341	429.237
1	Top	-28674.7	-301.756	321.928
2	Bottom	2500.183	-1942.38	-321.924
2	Top	1666.789	-1294.92	-214.618
3	Bottom	-5895.79	-896.928	788.334
3	Top	-2947.89	-448.464	394.167
4	Bottom	-5776.95	-299.581	-394.167
4	Top	5.69E-13	2.82E-14	7.21E-14
5	Bottom	5.69E-13	2.82E-14	7.21E-14
5	Top	5776.953	299.581	394.167
6	Bottom	2947.897	448.464	-394.167

6	Top	5895.795	896.928	-788.334
7	Bottom	-1666.78	1294.925	214.618
7	Top	-2500.18	1942.380	321.928
8	Bottom	28674.73	301.756	-321.928
8	Top	38232.98	402.3419	-429.237

Now Ply level stress and strains are calculated from Autodesk Heliu Composite Software are as follows.

Table 5: Ply level Strain values in MPa

Ply No	Position	ϵ_1	ϵ_2	γ_{12}
1	Bottom	-0.25689	0.024693	0.07896
1	Top	-0.21233	0.025671	0.067895
2	Bottom	0.02223	-0.18996	-0.06789
2	Top	0.11762	-0.15662	-0.03800
3	Bottom	-0.15146	-0.10236	0.143569
3	Top	-0.01866	-0.07562	0.071569
4	Bottom	-0.05631	-0.05203	-0.07156
4	Top	5.89E-18	2.3E-18	2.62E-17
5	Bottom	5.89E-18	2.3E-18	2.62E-17
5	Top	0.039455	0.123656	0.071569
6	Bottom	0.022258	0.035698	-0.07156
6	Top	0.045692	0.075213	-0.14359
7	Bottom	-0.01963	0.125896	0.038000
7	Top	-0.02563	0.218523	0.067895
8	Bottom	0.139569	-0.01333	-0.06789
8	Top	0.265359	-0.02789	-0.07896

Table 6: Ply Level Stress values in MPa

Ply No	Position	σ_1	σ_2	τ_{12}
1	Bottom	-35269.5	-405.666	413.263
1	Top	-30479.3	-299.956	343.790
2	Bottom	2725.86	-1789.25	-343.726
2	Top	1956.453	-1400.26	-246.447
3	Bottom	-8459.57	-952.23	788.334
3	Top	-4173.26	-398.33	394.167
4	Bottom	-6113.56	-249.99	-394.167
4	Top	6.38E-13	3.15E-14	6.33E-14
5	Bottom	6.38E-13	3.15E-14	6.33E-14
5	Top	6113.56	249.99	394.167
6	Bottom	4173.26	398.33	-394.167
6	Top	8459.57	952.23	-788.334
7	Bottom	-1956.45	1400.26	246.447
7	Top	-2725.86	1789.25	343.726
8	Bottom	30479.3	299.956	-343.790
8	Top	35269.5	405.66	-413.263

5. CONCLUSION

From this study we say that the results given by the analytical solver in Excel and Matlab gives nearly same results, and can be used for further work Ply level strain values obtained from Analytical and Experimental analysis follows the same pattern. The positive and negative values are for compressive and tensile values. Maximum strains occur at ply 4 top and ply 5 bottom. Strain increases from ply 1 to up to ply 4 and then decreases from ply 5 to ply 8.

For ply level stress analysis the values follows the same pattern as strain. Minimum stress occurs at ply 4 and 5, and stress increases from ply 4 to 1 and ply 5 to 8. Maximum stress occurs at ply no 8 and 1 but ply 1 is in compression and ply 8 is in tension. As we know composites are better in compression and fails in tension. Hence in Failure criteria analysis when Maximum stress failure criteria is applied to composite beam it fails at ply 8 due to transverse tensile stress. Failure mode of composite is Transverse failure. From Finite Element Analysis of Composite beam structure, the results obtained are deflection 1.77mm and stress values is 1731 MPa and results obtained from analytical solver are Deflection 1.859 mm and stress is 1780.56 MPa. The percentage error between these two values is 4.78% for Deflection and 2.75% for Stress, which is in the acceptable range. This shows that results obtained are correct and validates each other.

This study further can be expanded for Optimization of Composite Structure such as Plate, Beam, and Shell etc. All the solver we developed can be used for the that study and can be prove helpful in Optimization problems

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